Forecasting methods applied to engineering management

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Abstract. This paper presents arguments for the usefulness of a simple forecasting application package for sustaining operational and strategic engineering management and provides the description to create it. Section 1 contains the clarification of some basic terms and concepts for the present paper, like the importance of management seen as a package of essential decision-making instruments for the engineering sciences. In section 2 criteria for selecting the appropriate forecasting method for a studied problem are presented. Eight forecasting methods are proposed for implementing, as well. In section 3 forecast quality indicators are discussed. MATLAB is proposed as a programming platform for the forecasting instrument application.

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1 Introduction

The growing complexity and diversity of any business environment permanently generates problems, with solutions based on making and applying decisions. The limited character of material, financial and human resources implies the responsibility for reaching the objectives by finding the most favorable resource allocation and usage conditions. This is why almost all the management science disciplines propose efficient decision making methods.

Forecasting is mainly used in the "alternative analysis" and "result evaluation" steps. The most efficient approach is research and analysis completed with experience. Much more cheaper than experimenting, research and analysis' main characteristic is that it can deliver a model to simulate the problem. The mostly used solution is simulating the problem's variables in mathematical terms and relations, forecasting being here a very useful instrument.

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2 Proposed Forecasting Methods

It is generally accepted that the following criteria should be studied when choosing a forecasting method - see also Zäpfel (1996, p. 96): the accuracy of the forecast; the fineness of reactions; the stability; the calculation time and the memory needed therefore; the ways of influencing the process by the user.

Tempelmeier (1992, p. 34-105) proposes the following time series - forecasting method associations: for time series with no seasonal variations with a relatively constant level - gliding averages and first order exponential smoothing; with an increasing or decreasing trend - linear regression, second order exponential smoothing or Holt's method (modified second order exponential smoothing). For time series with seasonal variations the best suited methods are the decomposition of time series (Ratio-to-Moving-Average), Winters' method and the multiple linear regression. Other classifications can be also found in forecasting-related publications.

The calculus of *linear regression* is a statistical method used to quantify the functional link between a dependent variable and one (or more) independent variables. As a special case of multiple regressions, the linear regression is suitable for approximating the evolution of a time series, represented as linear.

Let A_t be the observed time series, with n elements ($t = \overline{1,n}$). One can write, if the time series is almost linear: $A_t = b_0 + b_1 t + \epsilon_t$

Using linear regression, the coefficients b_0, b_1 can be calculated, so that the norm of the vector $(\epsilon_1, \epsilon_2, ..., \epsilon_n)$ is minimal. In conclusion, at a certain moment, the forecasting equation is: $F_k = b_0 + b_1 k$.

The *weighted averages* method uses the weighted average of the last n periods from the available time series. The value n is defined during the decision process, normally being a small value, 3 or 4. The name "mobile" originates in the fact that the considered data set shifts/glides along the time series. The elements' weights can be equal for all periods, or also different. The condition is that the sum of the weights be equal to 1. If we suppose that the number of elements of the time series A_i is N, then the mobile averages also form a time series F_i with $N - n$ elements, according to the following:

$$F_{k+1} = \sum_{i=k-n-1}^{k} p_i A_i \text{ where } n < k \leq N, \sum_{i=1}^{n} p_i = 1$$

Here p_i are the weights, A_i is the time series and F_i are the forecasted values.

The *exponential smoothing* method is more complex than the mobile averages method. Here the deviations of more recent values have a weight higher than the errors from a more distant past. In practice, all values older than the forecasted value are taken into account. This method is characterized by a relative ease of calculus and by a reduced number of pre-calculated values for the current forecast. The formula for exponential smoothing is: $F_t = \alpha A_{t-1} + (1 - \alpha) F_{t-1}$ where F_t is the forecast for period t, A_{t-1} is the value of the time series element from the previous period, F_{t-1} the forecast for the previous period and $\alpha \in [0; 1]$ the smoothing coefficient. To initialize F_{t-1} the value of A_{t-2} is used. For α values like 0.1 or 0.2 are usually used. Higher values of α would mean giving a higher weight to recent elements of the time series, smaller values of α would assure higher weights for "older" values.

The software package should also contain the *modified double exponential smoothing*, also known as "Holt's method". This method is suitable especially for very dynamic data sets, without seasonal variations. In practice, the double exponential smoothing means using the first order exponential smoothing twice - the first order exponential smoothing applied to the time series of the first order averages (obtained using the method described above). This way weighted averages of the first order averages are obtained.

$$S'_{t-1} = \alpha A_{t-1} + (1-\alpha)S'_{t-2}; \quad S''_{t-2} = \beta S'_{t-2} + (1-\beta)S''_{t-3}$$

From the upper two equations we obtain: $F_t = 2S'_{t-1} - S''_{t-2}$ where F_t is the forecast for period t, A_{t-1} the value of the time series element from the previous period, S'_t the first order average at the end of period t, S''_t the second order average at the end of period t and α and β the smoothing coefficients. For initializing A_1 the value $S'_2 = S''_1 = A_1$ is used. Normally α and β can be freely chosen and usually are values like 0.1 or 0.2.

The *adaptive exponential smoothing* is best suited for forecasts based on a large set of data, without seasonal influence. In this case the smoothing coefficient α is calculated using the forecast error from the previous steps, according to the formula: $\alpha_{t+1} = \frac{|E_t|}{|M_t|}$ where $E_t = \beta(A_t - F_t) + (1+\beta)E_{t-1}$ and $M_t = \beta|A_t - F_t| + (1-\beta)E_{t-1}$.

Hence, the forecasting equation is $F_{t+1} = F_t + \alpha_t(A_t - F_t)$. The notations are the same as in the methods described before. The modifiable variable is α, with values usually around 0.1 or 0.2.

In many cases the granularity of the time series is not sufficient to have a complete view of the "past" used as a base for the forecast. For some data sets with sudden variations in short periods, undocumented by the values of the time series, a fractal interpolation is suitable to point out the potential variations. A forecast can be made using the new time series applying one of the classic forecasting methods, or the process can be further refined using *fractal interpolation.* The simplest way to interpolate a function $x(t)$, when the points $(t_i, x_i), i = 0, 1, \ldots, N$ are known starts with a "system of iterated functions".

$$W_n\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & 0 \end{pmatrix}\begin{pmatrix} t \\ x \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix}$$

where the coefficients a_n, c_n, e_n and f_n are determined from the following conditions, for n = 1, 2, . . . , N

$W_n\begin{pmatrix} t_0 \\ x_0 \end{pmatrix} = \begin{pmatrix} t_{n-1} \\ x_{n-1} \end{pmatrix}$, $W_n\begin{pmatrix} t_N \\ x_N \end{pmatrix} = \begin{pmatrix} t_n \\ x_n \end{pmatrix}$, the result being the following calculus equations:

$$W_n(t) \equiv t' = \frac{(t-t_0)}{(t_N-t_0)}t_n + \frac{(t-t_n)}{(t_0-t_N)}t_{n-1}$$

$$W_n(x) \equiv x' = \frac{(t'-t_{n-1})}{(t_n-t_{n-1})}x_n + \frac{(t'-t_n)}{(t_{n-1}-t_n)}x_{n-1}$$

in which $W_n(x) = x'$ is determined by a linear interpolation function (in t) between the points (t_{n-1}, x_{n-1}) and (t_n, x_n).

For better emphasizing the variations in a time series, a *variant of the fractal interpolation algorithm* has been developed, which fractal interpolates a time series using a geometric method. This method copies the initial form of the time series on every interval of the initial time series, keeping the geometric proportions of the initial sections, and can be also implemented.

In statistics, an *autoregressive integrated moving average (ARIMA)* model is a generalization of the autoregressive model with a mobile average (ARMA). These models are adapted to the time series to better understand the data or to make a forecast on the future points of the time series. The model is generally referred to as ARIMA(p,d,q), where p, d and q are natural numbers and represent the order of the autoregressive part (p), the integrated part (d), and the mobile average part (q). For a time series X_t (where t is an integer and X_t are real numbers) an ARMA(p,q) model is given by:

$$\left(1-\sum_{i=1}^{p}\phi_i L^i\right)X_t=\left(1+\sum_{i=1}^{q}\theta_i L^i\right)\epsilon_t$$

where L is the "lag" operator, ϕ_i are the parameters of the autoregressive part of the model, θ_i are the parameters of the moving average part and ϵ_t are errors. The errors ϵ_t are generally supposed to be independent variables, identically distributed according to a normal distribution with zero mean: $\epsilon_t \sim N(0,\sigma^2)$, where σ^2 is the variance. The ARMA model is generalized by adding a parameter d to form the ARIMA(p,d,q) model

$$\left(1-\sum_{i=1}^{p}\phi_i L^i\right)(1-L)^d X_t=\left(1+\sum_{i=1}^{q}\theta_i L^i\right)\epsilon_t$$

where d is a positive integer (if d is zero, the model would be equivalent to an ARMA model). The fact that not all parameter selections lead to good models has also to be taken into consideration. Particularly, if the model has to be stationary, then the parameters have to respect the conditions. Of course, there are also a few well-known special cases, for instance an ARIMA$(0,1,0)$ model given by $X_t = X_{t-1}+\epsilon$ is a simple "random walk". A number of variations of the ARIMA model are used for various applications.

In econometrics, an ARCH (autoregressive conditional heteroskedasticity) model, elaborated by Engle in 1982, considers the variance of the term "error" as a function of the variance of the errors of the previous periods. The ARCH method compares the error's variance with the square of the previous period's errors and is often used to model financial time series which present variable volatility in time. Concretely, let ϵ_t be the revenues (or revenue residues, the net value of a medium process) and let's assume that $\epsilon_t = \sigma_t z_t$, where $z_t \sim iid(0,1)$ and where the series σ_t^2 are modeled by

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \ldots + \alpha_p\epsilon_{t-p}^2, \alpha_0 > 0 \text{ and } \alpha_i \geq 0, i > 0.$$

If we presume the existence of an ARMA model for the error's variance, then the model is a GARCH model (*generalized autoregressive conditional heteroskedasticity*, Bollerslev (1986)). In this case, the GARCH(p,q) model is given by $\sigma_t^2 = \alpha_0+\alpha_1\epsilon_{t-1}^2+\ldots+\alpha_p\epsilon_{t-p}^2+\beta_1\sigma_{t-1}^2+\ldots+\beta_q\sigma_{t-q}^2$.
Generally, when heteroskedasticity is tested in econometric models, the best test is the "white test". However, in the case of time series, the best test is Engle's ARCH test.

3 Implementing the Forecasting Tools

Programs written for MATLAB run on various operating systems, therefore it is the ideal environment to create a forecasting application with didactic purpose, as well as for small enterprises to create proprietary forecasting tools for their specific needs.

To assure the quality of a forecast evaluating the qualities of the used forecasting methods may prove useful - before and during the calculus process, as well. We define as a forecasting error the difference between the real value A_t achieved in period t and the forecasted value F_t $(e_t = A_t - F_t)$.

The forecasting error isn't a useful measure for classifying the effectiveness of a forecasting method, but the sum of errors offers information on the polarization (*BIAS*) of the forecasted values. Many other error supervision alternatives are also used to calculate the level and dispersion of forecasting errors. This way, the calculus of the Mean Absolute Deviation (*MAD*) is used to quantify the dispersion and thus the credibility of the method; the Mean Squared Error (*MSE*) is used to emphasize the mean "distance" of the forecasted values compared to the real values and the Coefficient of Variance (CV) is useful for easing the comparison of different data sets' forecasts. For a good forecasting method the error values should fluctuate around zero - therefore a tracking signal can provide information on that:

$$SIG_t = \frac{\sum_{i=1}^{t} e_i}{MAD_t} = n\frac{\sum_{i=1}^{t} e_i}{\sum_{i=1}^{t} |e_i|}$$

A good alternative to the software written for MATLAB is FreeFore (Autobox), which can be used for comparison purposes. The solution offered by FreeFore is a time series analysis instrument, whose results can be used to choose one of the 15 implemented forecasting methods. Still, FreeFore allows "manually" selecting the method to be used.

4 Conclusion

The proposed set of forecasting methods has been implemented using MATLAB, tests have been carried out using various data sets, such as retail stock data presenting seasonal variations, daily exchange rates on a long term and sales volumes with a trend and no seasonal variations. The values forecasted coincided in most cases with the ones provided by FreeFore, the free software using the Autobox forecasting engine. This is a positive argument for implementing forecasting software with a didactic purpose during the management education of engineers. Creating an automated forecasting model could be a useful extension of the proposed software package. As for small engineering enterprises, self-programmed software on the base of MATLAB can provide flexibility of the application at a low cost.

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